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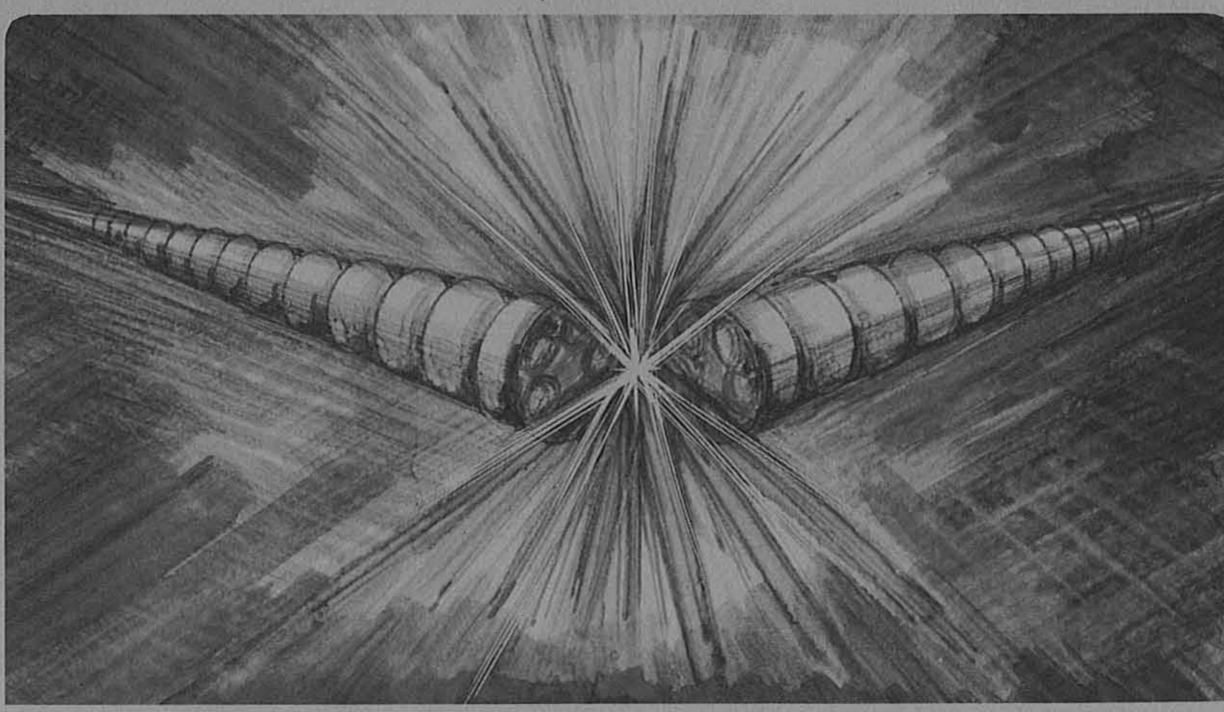

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The Evolution and Limits of Spectral Bandwidth in Free Electron Lasers*

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Abstract

We study the bandwidths in free electron lasers(FELs) under different circumstances; For weakly saturated FELs in storage rings, the bandwidth is given by the formula derived in the super mode theory, while it is given by the Fourier tranform of the electron pulse length in the strongly saturated FELs in linacs. The limiting bandwidth for the case of a DC beam is given by a Schawlow-Townes formula, but the approach to the limit is very slow.

1. Introduction

There has been some confusion on the achievable bandwidth in free electron laser (FEL) oscillators[1]. One often hears about the transform limited bandwidth, which is the bandwidth limited by the Fourier transform of the electron beam. In the supermode theory, the bandwidth is given by a geometric average of the gain bandwidth and the transform limited bandwidth [2], [3], [4]. Extension of the Schawlow-Townes limit [5] to FEL has also been discussed [6]. In this paper, we study the evolution of the spectral and the temporal profile in FELs in terms of a simple but physically reasonable model introduced in section 2, and determine the circumstances under which different bandwidth formula are applicable.

We show in section 3 that the bandwidth of the optical pulse narrows as $1/\sqrt{n}$ as the number of the passes n of the electron beam through the optical cavity increases [7]. The temporal width also narrows in a similar fashion in the beginning of the intensity build-up. This, together with the fact that the product of the temporal and the spectral width must be greater than a minimum value, leads to the limiting bandwidth predicted by the super mode theory. The supermode theory is valid for a weakly saturated system such as storage ring based FELs, where the gain can be regarded as a constant.

For linac based FELs discussed in section 4, however, the optical power evolves to a level where the reduction of gain due to high intensity,i.e., the gain saturation, becomes important. Observing that the gain saturation is homogeneous in frequency but inhomogeneous in time, we derive that the limiting bandwidth is then given by the Fourier transform of the electron pulse length. In this discussion, we find it necessary to distinguish between the intensity saturation from the " spectrum sauration"; The time to reach the limiting spectrum is typically longer than the time to reach the intensity saturation.

For the case of a DC electron beam, the narrowing of the bandwidth continues until it reaches a small value determined by the spontaneous radiation. However, as the bandwidth narrowing is slow, $1/\sqrt{n}$, it takes a long time to reach this value, typically a day or longer. This is discussed in section 5.

2. Equation for FEL Evolution

We consider the evolution of the optical signal in a FEL cavity. The increase of the optical power during the nth passage of the electron beam consists of two terms, that due to the amplification of the power already present and that due to the spontaneous radiation. Let $dP(\omega,\tau;n)/d\omega$ be the τ-dependent spectral density of the optical power at the beginning of the nth passage, and $dS(\omega,\tau)/d\omega$ a similar quantity due to the spontaneous radiation emitted in one pass. Here, ω is the frequency and $c\tau$ (c = speed of light) is the distance from the pulse center. Notice that ω and τ are conjugate variables under Fourier transformation. Thus, the quantity $dP(\omega,\tau;n)/d\omega$ should be, strictly speaking, understood as the Wigner distribution [8]. However, it can be loosely interpreted as the spectral density evaluated at τ, when the following inequality is valid :

$$\sigma_\omega \sigma_\tau \geq \frac{1}{2} . \tag{1}$$

Here σ_ω and σ_τ are the rms values of the spectral and the temporal widths, as follows:

$$\sigma_\omega{}^2 = \int (\omega - \omega_0)^2 \frac{dP}{d\omega} d\omega d\tau \Big/ \int \frac{dP}{d\omega} d\omega d\tau , \tag{2}$$

$$\sigma_\tau{}^2 = \int \tau^2 \frac{dP}{d\omega} d\omega d\tau \Big/ \int \frac{dP}{d\omega} d\omega d\tau , \tag{3}$$

Here, ω_0 is the central frequency.

With this interpretation, a simple model for the evolution of the optical power may be written as follows:

$$\frac{d}{dn}\left(\frac{dP(\omega,\tau;n)}{d\omega} \right) = (g\,(\omega,\tau;n) - \alpha) \frac{dP(\omega,\tau;n)}{d\omega} + \frac{dS}{d\omega} \tag{4}$$

Here g is the gain parameter and α is the total loss per round trip.

To solve the evolution equation, we need to specify the behavior of the gain function $g(\omega,\tau;n)$. We consider the cases of storage rings and linacs separately.

3. Supermode Bandwidth in Storage Ring Based FELs

In storage ring based FELs, the saturation is due to the induced energy spread and bunch lengthening of the electron beams that accumulate from pass to pass. The power level at saturation is determined by a balance between the inhomogeneous gain reduction and radiation damping, and given by the Renieri limit [9]. It is well below the level at which particle trapping in the ponderomotive potential becomes significant. Therefore, we may assume that the gain is independent of the optical power and n, as follows:

$$g(\omega,\tau;n) = g_0 F(\omega) T(\tau) \,. \tag{5}$$

The function $F(\omega)$ describes the frequency dependence of the gain. For frequencies near the resonance frequency ω_0,

$$F(\omega) = 1 - \frac{(\omega-\omega_0)^2}{2\sigma_N^2\omega_0^2} \quad ; \quad \frac{|\omega-\omega_0|}{\omega_0} < \sigma_N. \tag{6}$$

In the above, σ_N is the gain bandwidth, given approximately by

$$\sigma_N \sim \frac{1}{2N} \,. \tag{7}$$

where N is the number of the undulator periods.

The function $T(\tau)$ describes the temporal profile of the electron pulse: For τ near the pulse center, it is of the form

$$T(\tau) = 1 - \left(\frac{\tau}{2\sigma_\tau^e}\right)^2 \,. \tag{8}$$

Here σ_τ^e is the rms bunch length of the electron pulse in time.

With the gain function specified by Eq. (5), Eq. (4) can be solved easily. The result is:

$$\frac{dP(\omega,\tau;n)}{d\omega} = \frac{\mathrm{Exp}[(g_0F(\omega)T(\tau)-\alpha)n]-1}{g_0F(\omega)T(\tau)-\alpha}\frac{dS}{d\omega} \tag{9}$$

In view of Eqs. (6) and (8), Eq. (9) implies that the spectral width and the temporal width of the optical pulse (defined by Eqs. (2) and (3)) become narrower as the number of passes n increases as follows:

$$\frac{\sigma_\omega}{\omega_0} = \sigma_N\sqrt{\frac{1}{g_0n}} \quad , \tag{10}$$

$$\sigma_\tau = \sigma_{\tau e}\sqrt{\frac{1}{g_0n}} \quad . \tag{11}$$

The simultaneous narrowing in the spectral width Eq. (10) and in the temporal width, Eq. (11), must stop to be consistent with the inequality (2). This occurs for

$$n \geq n_c = \frac{2\pi c\sigma_\tau^e}{g_0\lambda N} \quad . \tag{12}$$

The limiting bandwidth in this case is

$$\frac{\sigma_\omega}{\omega} = \sqrt{\frac{1}{2N}\frac{\lambda}{4\pi c\sigma_\tau^e}} \tag{13}$$

This is a geometric average of the gain bandwidth and the transform limited bandwidth discussed in the following section. Equation (13) was first derived in the context of the super mode theory[2]. The simultaneous narrowing of the spectral and the temporal widths was discussed in Ref. [3].

Equation (13) appears to be consistent with the results of FEL experiments in storage rings [3], [4] (with a suitable replacement of 1/2N by a factor appropriate for optical klystrons).

4. Bandwidth limited by Electron Beam Pulse Length in Linac-Based FELs

In linac driven FELs, the optical pulse interacts with a fresh electron bunch in each round trip. The particle trapping in the ponderomotive potential becomes significant, and the FEL intensity reaches saturation when the electron motion in the undulator corresponds to about one half of the synchrontron oscillation period. In this case, it is necessary to take into account the gain reduction caused by high intensity effect. A simple way to model the gain reduction is to replace Eq. (5) by:

$$g(\omega,\tau;n) = \frac{g_0 F(\omega) T(\tau)}{1 + P(\tau; n)/\overline{P}} \quad , \tag{14}$$

$$P(\tau; n) = \int \frac{dP(\omega,\tau;n)}{d\omega} d\omega \quad . \tag{15}$$

In Eq. (14), $\overline{P}$ is a parameter which sets the scale of the saturation intensity; it is about the power at which electrons undergo a one-half period synchrotron oscillation in passing through the undulator. According to Eq. (14), the gain reduction for a given frequency ω and temporal position τ is determined by a sum of the optical intensities over all ω, but evaluated at the same τ. Thus, Eq. (14) is a model for a gain saturation which is homogeneous in ω but inhomogeneous in τ [10]. The gain saturation is homogeneous in ω because all frequency components which lie within the gain bandwidth should contribute equally to the saturation. [Strictly speaking, the integral in Eq. (15) should be replaced by an integral which extends to frequency values separated by a gain bandwidth from ω. However, we are mainly interested in the cases for which the spectral width of the function $dP/d\omega$ is much narrower than the gain bandwidth, in which case Eq. (15) is a good approximation]. On the other hand, the saturation is inhomogeneous in τ since optical intensities at two τ's separated by more than the slippage distance $N\lambda/c$ should evolve independently [11]. Here we are assuming, as as almost always the case, that the pulse is much longer than the slippage distance.

The evolution of the optical pulse profiles in linac driven FELs can therefore be determined by explicitly solving Eqs. (4) and (14). However, the main feature of the spectral property can be determined qualitatively as follows:

In the beginning of the FEL evolution the ratio $P(\tau;n)/\overline{P}$ is small so that Eq. (14) reduces to Eq. (5). Therefore, the spectral width and the temporal width will both start to narrow as described by Eq. (10) and Eq. (11), respectively. As the optical power increases, the gain becomes smaller due to the intensity dependent effect. The saturation takes place first at $\tau=0$ where the initial gain is highest. However, the optical intensities at $\tau\neq 0$ will keep increasing until they reach their own saturation level. Thus, the temporal width of the optical pulse, after initial narrowing, will broaden as the optical intensity approaches the saturation level, and eventually becomes the same as the width of the electron beam.

The limiting bandwidth in this case is therefore obtained from Eq. (2) by replacing σ_τ by σ_τ^e:

$$\frac{\sigma_\omega}{\omega}=\frac{\lambda}{4\pi c\sigma_\tau^e} \quad . \tag{16}$$

Equation (16) is often referred to as the Fourier-transform-of-the-electron-pulse limited, or simply the transform limited bandwidth. In terms of the full-width-at-the-half-maximum (FWHM) quantities, Eq. (16) becomes

$$\left(\frac{\Delta\omega}{\omega}\right)_{FWHM}\geq\frac{0.44\,\lambda}{c(\Delta\tau)_{FWHM}} \quad . \tag{17}$$

Equation (16) or (17) is consistent with the result of the FEL experiments in linacs [12].

For explicit solution of Eq.(14), we proceed as follows: We integrate Eq. (4) with respect to ω. In doing so, we assume that the width of the optical spectrm is much narrower than the gain bandwidth so that

$$\int d\omega\, F(\omega)\frac{d\,P(\omega,\tau;n)}{d\omega}=\int\frac{d\,P(\omega,\tau;n)}{d\omega}\,d\omega=P(\tau;n). \tag{18}$$

The result of integration is

$$\frac{d}{dn}P(\tau;n)=\left(g(\tau;n)-\alpha\right)P(\tau;n)+\Delta S \quad , \tag{19}$$

where $\Delta S = \int d\omega(dS/d\omega)$ is the total spontaneous power,

$$g(\tau;n) = \frac{g_0(\tau)}{1+P(\tau;n)/\overline{P}}; \quad g_0(\tau) = g_0 T(\tau) . \tag{20}$$

In solving Eqs. (19) and (20), we introduce the dimensionless parameter

$$\varepsilon = \frac{\Delta S}{P} \quad , \tag{21}$$

which is a very small number, typically 10^{-8} or less. If $g_o(\tau) < \alpha\text{-}\varepsilon$, the FEL is below threshold, and $P(n,\tau)$ is of the order ΔS for all n. On the other hand, if $g_o(\tau) > \alpha\text{-}\varepsilon$, $P(n,\tau)$ evolves to a saturated value

$$P_S = \frac{g_0(\tau) - \alpha}{\alpha}\overline{P} . \tag{22}$$

In this case the solution of Eq. (20) is

$$\frac{(P(\tau;n)+P_0(\tau))^{x-1}}{(P(\tau;n) + P_s(\tau))^{x+1}} = \frac{(P_0(\tau))^{x-1}}{(P_s(\tau))^{x+1}} e^{2\alpha n} , \tag{23}$$

where

$$x = \frac{g_0(\tau)+\alpha}{g_0(\tau)-\alpha} \text{ and } P_0(\tau) = \frac{\Delta S}{g_0(\tau)-\alpha} \quad . \tag{24}$$

The behavior for the limiting cases of n are

$$P(\tau;n) = P_0(\tau)(e^{(g_0(\tau)-\alpha)n} - 1); \; n<<n_s , \tag{25}$$

$$P(\tau;n) = P_s(\tau)[1 - (P_s(\tau)/P_0(\tau))^{\alpha/g_0(\tau)}e^{-\alpha(1-\alpha/g_0(\tau))n}]; \; n>>n_s \tag{26}$$

In the above, n_S is the number of passes characterizing the saturation of the power; The optical power $P(\tau;n)$ is practically constant at $P_S(\tau)$ for $n \geq n_S$. The quantity n_S is given by

$$n_s = \frac{1}{g_o(\tau)} \ln\left(\frac{P_s(\tau)}{P_o(\tau)}\right) \quad . \tag{27}$$

From Eq. (25), we see that the temporal width begins to narrow at small n. However, at saturation, the temporal profile is, assuming that $g_0 >> \alpha$, given by $g_0(\tau) = g_0 T(\tau)$ from Eq. (22), and is the same as that of the electron pulse. The limiting bandwidth is then given by Eq. (16).

Having determined the function $P(\tau;n)$, and therefore $g(\tau;n)$, Eq. (4) can be integrated straightforwardly. The behavior of $dP/d\omega$ at large n is approximately the same as that of the solution of the homogeneous equation

$$\frac{dP(\omega,\tau;n)}{d\omega} = \frac{dP(\omega,\tau;0)}{d\omega} e^{G(\tau;n)F(\omega)-\alpha n} , \tag{28}$$

where

$$G(\tau;n) = \int_0^n g(\tau;n)dn \quad . \tag{29}$$

The spectral distribution given by Eq. (28) is a Gaussian shape, with the rms relative width

$$\frac{\sigma_\omega}{\omega_o} = \sigma_N \sqrt{\frac{1}{G(\tau;n)}} \quad . \tag{30}$$

We can derive from Eqs (19), (25) and (26) that

$$\begin{aligned} G(\tau;n) &\sim g_0 n \quad \text{for } n<<n_s \ , \\ &\sim \alpha n \quad \text{for } n>>n_s \ . \end{aligned} \tag{31}$$

We have therefore

$$\frac{\sigma_\omega}{\omega_0} = \sigma_N \sqrt{\frac{1}{g_0 n}} \ ; \ n<<n_s \ , \tag{32}$$

$$= \sigma_N \sqrt{\frac{1}{\alpha n}} \; ; \; n >> n_s \; . \tag{33}$$

The bandwidth will keep narrowing as described by Eq. (32) and (33) until it reaches the Fourier transform (of the electron beam) limit given by Eq. (16). Typically, the bandwidth after n_s passes ($\sim \sigma_N/\sqrt{g_0 n_s}$) is still broader than the Fourier transform limit. Therefore the spectrum of optical pulses in a FEL cavity keeps evolving after the intensity reached saturation at around $n=n_s$. From Eqs. (16) and (33), the number of passes n_ω required to reach "spectrum" saturation is

$$n_\omega = \frac{1}{\alpha}\left(\frac{4\pi c \sigma_\tau^e}{2N\lambda}\right)^2 . \tag{34}$$

5. Intrinsic Limit Due to Noise

The bandwidths formula, Eqs. (13) and (16), are applicable when the electron pulse length is finite. We now consider the case of a DC electron beam. Thus, we delete the τ-dependence in Eq. (4), and replace $g(\omega,\tau;n)$ by $g(n)F(\omega)$. The steady state solution is obtained by setting the R.H.S. of Eq. (4) to zero;

$$\lim_{n \to \infty} \frac{dP(\omega;n)}{d\omega} = \frac{dP_s(\omega)}{d\omega} = \frac{1}{\alpha - \bar{g}F(\omega)} \frac{dS}{d\omega} \tag{35}$$

In the above, we have deleted the τ dependence because the solution is independent of τ. We consider a small neighborhood of ω near ω_0, so that the expansion Eq. (6) is valid. In Eq. (35), $\bar{g}$ is the limiting value of $g(n)$ for a large n which must be very nearly equal to α. Thus, we write

$$\bar{g} = \alpha\left(1 - \frac{\delta^2}{2}\right), \tag{36}$$

where $\delta << 1$. The δ^2 term enters with a negative sign so that Eq. (35) is positive definite for all ω. Inserting Eq. (36) to (35), we obtain

$$\frac{dP_s}{d\omega} = \frac{1}{\alpha\left[\frac{\delta^2}{2} + (\omega-\omega_0)^2/2\sigma_N^2\omega_0^2\right]}\left(\frac{dS}{d\omega}\right) . \tag{37}$$

By integrating Eq. (37) (neglecting the ω-dependence of $dS/d\omega$), we obtain

$$\delta = 2\pi \frac{\Delta S}{P_{gen}}\ ;\ \Delta S = \sigma_N \omega_0 \frac{dS}{d\omega} \approx \int d\omega \frac{dS}{d\omega}\ , \tag{38}$$

where $P_{gen} = \alpha P_s$ is the generated power. Thus, the limiting distribution is a Lorentzian with

$$\left.\frac{\Delta\omega}{\omega}\right|_{FWHM} = 2\pi\sigma_N \frac{\Delta S}{P_{gen}}\ . \tag{39}$$

Equations (37) and (39) are similar to the Schawlow-Townes formula [5] except for the replacement of the bandwidth of the optical cavity by the gain bandwidth σ_N. Typically, the limiting bandwidth is smaller at least by a factor 10^{+6} compared to σ_N. To reach this bandwidth via the gain narrowing described by Eq. (33), it will take at least 10^{12} passes, which corresponds to about one day with a 10-m optical cavity. A single mode operation of an FEL with a bandwidth similar in magnitude to that given by Eq. (39) has been reported [13]. However, the result is controversial experimentally and unlikely theoretically because of the slow approach to the limiting bandwidth. The approach to the frequency saturation in long pulse FELs was also studied in ref. [14], where the statistical effect of the spontaneous radiation is included.

Acknowledgement

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References and Footnotes

[1] Round table discussion chaired by K.-J. Kim at Tenth International Free Electron Laser Conference, Jerusalem, Israel, August 29 - September 2, 1988.

[2] G. Dattoli and A. Renieri, Nuovo Cim. 59B (1980) 1; G. Dattoli, A. Marino, A. Renieri and F. Romanelli, IEEE J. Quatum Electron. QE-17 (1981) 211.

[3] P. Elleaume, IEEE J. Quantum Electron. QE-21 (1985) 1012.

[4] V. N. Litvinenko and A. N. Vinokurov, "Lasing spectrum and temporal structure in storage ring FEL: Theory and experiment," these proceedings.

[5] A. L. Schawlow and C. H. Townes, Phys. Rev. 112 (1958) 1940.

[6] A. Gover, A. Amir and L. R. Elias, Phys. Rev. 35, (1987) 164.

[7] This behavior was pointed out by the author in several private discussions during and after the Seventh FEL conference at Lake Tahoe, California, September, 1985.

[8] E. Wigner, Phys. Rev. 40 (1932) 749; A. Walther, J. Opt. Soc. Am. 58 (1968) 1256; E. Wolf, J. Opt. Soc. Am. 68 (1978) 6; K.-J. Kim, Nucl. Instr. Meth., A246 (1986) 71.

[9] A. Renieri, Nuovo Cim., 53B (1979) 160.

[10] For a discussion of the homogeneous and inhomogeneous saturation in conventional lasers, see for example, A. Yariv, Optical Electronics, Holt, Rinehart and Winston, Inc., 1985, Chap. 5.

[11] The validity of Eq. (14) has been confirmed by numerical calculation. G. Dattoli, private communications.

[12] R. Warren, et al., Nucl. Instr. Method, A285 (1989) 1.

[13] L. Elias, G. Ramian and A. Amir, Phys. Rev. Lett. 75 (1986) 424.

[14] B. Levush and T. M. Antonsen, Nucl. Instr. Method., A285 (1989) 136; T. M. Antonsen, these proceedings.

Errata:

There are two typographical errors in Page 7 of the paper "The Evolution and Limits of Spectral Bandwidth in Free Electron Lasers".

(1) In the sentence above Eq. (22), $P(n,\tau)$ should be replaced by $P(\tau;n)$.

(ii) Equation (23) should read

$$\frac{(P_o(\tau) + P(\tau;n))^{x-1}}{(P_s(\tau) - P(\tau;n))^{x+1}} = \frac{(P_o(\tau))^{x-1}}{(P_s(\tau))^{x+1}} e^{2\alpha n} .$$